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AUTHOR(S)- L. P. Gieseler

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ABSTRACT

Equations and theory are developed from which it is possible to determine the lighting conditions at earth launch and lunar landing for any time during the lunar month, any season of the year, and any time during the lunar precession cycle of 18.6 years. A simplified sun-earth-moon model is considered, in which the lunar equator is assumed to lie in the plane of the lunar orbit, which is assumed to be circular. The equations are used to determine how Apollo launch opportunities are influenced by: 1) Requiring that the elevation of the sun at lunar landing be within the range from 7 to 20 degrees from the eastern horizon, and that the selenographic longitude of the landing site be within the range from -45 to +45 degrees, 2) Requiring that earth launch take place in daylight, and 3) Simultaneously imposing both of the above constraints.

A graphical technique is used in which restriction 1 is represented by one area on a graph, restriction 2 gives rise to another area on the same graph, and restriction 3 is then given by the intersection of the two areas. The results show that for the assumed lunar lighting constraint the vast majority of daylight launch windows are associated with Pacific injections for any month during the 18.6 year lunar cycle. The high correlation between daylight launches and Pacific injections is then explained geometrically by relating the position of the earth launch site for the two daily launch windows to the target earth-moon vector and the position of the sun.

The graphs are presented in such a way that the effect of different lunar lighting constraints can be evaluated. Also the effect of restricting launches to something other than the period from sunrise to sunset can be evaluated analytically from the equations and approximately from the existing graphs.

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BELLCOMM, INC.

SUBJECT: The Influence of Earth Launch and Lunar Lighting Constraints on the Apollo Mission - Case 310

DATE: May 26, 1967

FROM: L. P. Gieseler

TM-67-2013-4

TECHNICAL MEMORANDUM

I. INTRODUCTION

Current ground rules for the Apollo mission specify that the elevation of the sun from the eastern horizon be restricted to the range between 7 and 20 degrees at lunar landing. It is easy to see that for a single landing site this can only occur for approximately one day out of a solar month of 29.53 days. Neglecting lunar physical librations, the dawn terminator or sunrise line moves at the rate of $\frac{360}{29.53} = 12.2°$ per day. To an observer near the lunar equator the sun appears to move at the same rate across the sky. Therefore the time interval that the elevation of the sun falls within the required elevation range will be slightly more than one day, and the potential number of missions to that site is essentially reduced to one per solar month.

A further reduction in launch opportunities is produced if the requirement is invoked that launch from the earth take place in daylight. One might expect that on the average 50% of the launch opportunities would occur in daylight and 50% would occur at night. The actual percentage occurring in daylight is considerably less than this because any earth lighting constraint implies that an entire launch window should occur in daylight.

Earth launch lighting conditions are determined in this memorandum by first determining the inertial position of the launch site vector for different angular positions of the moon during the month. When in addition the angular position of the sun relative to the earth is specified, the lighting conditions for both earth launch and lunar landing can be computed. The results of computations of this type are presented in the form of contour graphs which use the above-mentioned positions of the sun and moon as coordinate variables. From the graphs and geometrical analysis, general conclusions are drawn about the manner in which the lighting constraints influence launch opportunities for an entire 18.6 year lunar cycle.

A list of symbols and their definition is given in Appendix I. Appendix II presents details of the computation of the launch site vector. Appendix III gives a geometrical explanation of some of the conclusions.

II. POSITIONS OF THE SUN, MOON, AND LAUNCH VECTOR

The apparent motion of the sun relative to the earth is shown in Figure 1a. QA_s is the angular position of the sun measured along the ecliptic and referenced to the vernal equinox. From spherical trigonometry,

$$RA_s = \tan^{-1}(\cos i_s \tan QA_s) \tag{1}$$

$$0 \leq RA_s \leq 360°$$

$$(RA_s \text{ is in the same quadrant as } QA_s)$$

$$DEC_s = \sin^{-1}(\sin i_s \sin QA_s), \quad -90° \leq DEC_s \leq 90° \tag{2}$$

where

RA_s = right ascension of the sun

DEC_s = declination of the sun

i_s = angle between ecliptic and equatorial planes ($\cong 23.45°$)

The motion of the moon relative to the earth is shown in Figure 1b. QA_m is the angular position of the moon in its orbit, measured from its ascending node. The position of the nodal line relative to the vernal equinox is specified by the angle RA_{node}, which varies from approximately -13 to +13 degrees in an 18.6 year cycle as shown in Figure 2. Referring to Figure 1b,

$$R_m = \tan^{-1}(\cos i_m \tan QA_m) \tag{3}$$

$$0 \leq R_m \leq 360°$$

(R_m is in the same quadrant as QA_m)

$$RA_m = R_m + RA_{node} \tag{4}$$

$$DEC_m = \sin^{-1}(\sin i_m \sin QA_m), \quad -90° \leq DEC_m \leq 90° \tag{5}$$

where

RA_m = right ascension of the moon

DEC_m = declination of the moon

i_m = inclination of the lunar orbit plane with respect to the equatorial plane. i_m varies between approximately 18.5 and 28.5 degrees in the same 18.6 year cycle as exhibited by RA_{node}. (See Figure 2)

As shown in Figure 1b, the spacecraft orbital plane is assumed to contain the earth-moon vector as determined at lunar arrival, and makes a dihedral angle DL with the lunar orbital plane. The inertial position vector of the launch site lies in the spacecraft orbital plane. For studies of the lighting conditions at earth launch it is desirable to determine the right ascension of the launch vector when the following quantities are given:

AL - launch azimuth

RA_m - right ascension of the moon at arrival

DEC_m - declination of the moon at arrival

ϕ_L - launch site latitude

RA_m and DEC_m then define the "target" earth-moon vector. Appendix II gives the appropriate equations and their derivation. As indicated there, two solutions are possible:

$$\begin{aligned} RAL_A &= f_1(AL, RA_m, DEC_m, \phi_L) \\ RAL_P &= f_2(AL, RA_m, DEC_m, \phi_L) \end{aligned} \tag{6}$$

where

RAL_A = right ascension of launch site which is ahead of the earth-moon vector (corresponds to an Atlantic injection)

RAL_P = right ascension of launch site which is behind the earth-moon vector (corresponds to a Pacific injection)

An understanding of the gross behavior of RAL_A and RAL_P as QA_m varies will be useful for the succeeding sections of this memorandum. Referring to Figure 1b, when $QA_m = 0$, RAL_A is ahead of the earth-moon vector by about 90°. As QA_m increases to 360° RAL_A remains ahead of the earth-moon vector by a varying amount which is restricted to the range 0° - 180°. In an analogous way, as QA_m varies from 0° to 360° RAL_P starts about 90° behind the earth-moon vector and remains behind it by a varying amount which is restricted to the range 0° - 180°.

To each of the two launch opportunities there corresponds a dihedral angle DL_A (Atlantic injection) and DL_P (Pacific injection). An in-plane launch is defined as the launch opportunity whose dihedral angle is the smaller of the two. As shown in Appendix II and Figure 21, Atlantic injection launches are out-of-plane and Pacific injection launches are in-plane when QA_m is in the range $90° \leq QA_m \leq 270°$. The situation is reversed during the remainder of the lunar cycle.

III. LIGHTING CONDITIONS AT LUNAR LANDING

The lunar orbital plane and the lunar equatorial plane are inclined to the ecliptic plane by approximately 5° and 1-1/2° respectively. These angles are small enough so that for the purposes of determining lunar lighting conditions all three planes can be considered coincident. Figure 3 shows the geometry involved. An observer is assumed to be stationed at point 0, on the moon's surface and on the earth-moon line. Ignoring longitudinal librations (assuming a circular lunar orbit), this corresponds to a selenographic longitude (λ_s) of zero. The elevation of the sun (E_{sm}) is defined as the angle between the moon-sun vector and a vector drawn from 0 to the eastern horizon. From the figure

$$A = QA_m' + 180 - QA_s$$

$$E_{sm} = A + D = A + 90°$$

Therefore,

$$E_{sm} = QA_m' - QA_s - 90° .$$

To determine QA_m', it is sufficiently accurate to neglect the fact that QA_m of Figure 1b lies in a different plane from RA_{node}. Then

$$QA_m' \cong QA_m + RA_{node}$$

and

$$E_{sm} \cong QA_m - QA_s + RA_{node} - 90° \qquad (7)$$

Figure 4 is similar to Figure 3, except that the observer is assumed to be stationed at a selenographic longitude equal to λ_s. It can be seen that

$$D = \lambda_s + 90° \quad ,$$

and

$$E_{sm} \cong QA_m - QA_s + RA_{node} - 90° + \lambda_s \tag{8}$$

Note that E_{sm} is 0 at sunrise, 90° when the sun is at the zenith, 180° at sunset, and 270° when the sun is at the nadir. For sunrise at the moon with $\lambda_s = RA_{node} = 0$, eq. (8) indicates that the moon should be 90° ahead of the sun. This can also be deduced directly from Figure 3.

IV. LIGHTING CONDITIONS AT LAUNCH

The lighting conditions at earth launch can easily be determined if the position of the sun relative to the earth (RA_s and DEC_s) and the inertial position of the launch site (RAL_A or RAL_P and ϕ_L) are known. The geometry involved is shown in Figure 5. The angular distance of the sun from the zenith is equal to the arc f. The elevation of the sun above the horizon (E_{se}) can be computed as follows:

$$F = RA_s - RAL \tag{9}$$

$$f = \cos^{-1}(\sin\phi_L \sin DEC_s + \cos\phi_L \cos DEC_s \cos F) \tag{10}$$

$$E_{se} = 90 - f \tag{11}$$

For the sunrise or sunset, $E_{se} = 0$ and equation (10) reduces to

$$F_o = \cos^{-1}(-\tan \phi_L \tan DEC_s) \tag{12}$$

At noon, $F = 0$, $RA_s = RAL$, and $E_{se} = 90 - \phi_L + DEC_s$.

To distinguish between sunrise and sunset, an approximate formula can be used, in which it is assumed that $DEC_s = \phi_L = 0$. Then $f = F$ and

$$E'_{se} = 90 - RA_s + RAL \tag{13}$$

Like E_{sm}, E'_{se} is 0° at sunrise, 90° at noon, 180° at sunset, and 270° at midnight.

The length of the longest day at Cape Kennedy can be determined by setting $\phi_L = 28.5°$, $DEC_s = 23.45°$, and computing the time required for the earth to rotate through an angle $2F_o$. Equation (12) gives $F_o = 103.62$ degrees. Then

$$D_L = \frac{2 \cdot 103.62}{w_e} = 13.8 \text{ hours} \tag{14}$$

where

D_L = length of the longest day at Cape Kennedy

w_e = angular velocity of the earth, relative to the sun (15°/hr).

Similarly, the length of the shortest day, (D_s) is determined by setting $DEC_s = -23.45°$ in which case $F_o = 76.38°$ and $D_s = 10.2$ hours.

V. RESULTS

(1) Graphical Representation

It is convenient to consider the quantities QA_s and QA_m as independent variables, and to use them as the coordinate axes in a graphical representation of the lighting conditions at earth launch and lunar landing. A change from 0 to 360° in QA_m represents one lunar month of 27.3 days. A similar change in QA_s represents one year of 365.25 days. When $QA_s = 0$, the sun is at the vernal equinox; when $QA_m = 0$, the moon is at its earth equatorial ascending node. Figure 6 is an example of the type of graph described. As time increases the moon moves upwards and to the right, tracing out successive parallel straight lines (ignoring lunar eccentricity) having a slope of approximately 13. The phase relationship between moon and sun must be known to locate the path exactly. The locus of the moon's motion as shown on Figure 6 was plotted from the following table:

Occurrence of Lunar Ascending Node			
Date	Time	Time from Equinox	Abscissa (QA_s)
April 8, 1967	20 h	18.5 days	18.23°
May 6, 1967	2	45.75	45.09°
June 2, 1967	9	73.04	71.99°
June 29, 1967	15	100.29	98.84°

The data for the first two columns was obtained from the American Ephemeris and Nautical Almanac for 1967. The time of occurrence of the vernal equinox is given there as March 21 at 8:00 a.m. The third column is the time interval from the equinox to the time of the lunar ascending node. This is converted in column 4 to degrees by multiplying by .9856, the average apparent angular velocity of the sun in degrees per day. Column 4 gives the values of QA_s for which QA_m equals 0 or 360°, and these are the points plotted in Figure 6. As mentioned before, a straight line relation between QA_m and QA_s is assumed.

(2) Lunar Lighting

In this section the restrictions imposed on launch opportunities by the lunar lighting constraint will be developed. Equation (8) is the governing equation. For plotting purposes it is convenient to introduce a new variable E_o which is defined by the equation:

$$E_o = E_{sm} - \lambda_s \tag{15}$$

Equation (8) then becomes:

$$E_o \cong QA_m - QA_s + RA_{node} - 90 \qquad (16)$$

Note that E_o is the elevation of the sun at $\lambda_s = 0$. Equation (16) is plotted in Figure 7 by allowing E_o to take on various fixed values as shown. The resulting graph is a family of straight lines from which the lighting conditions at a lunar landing site can be determined when QA_s, QA_m, and λ_s are known. ($RA_{node} \cong 0$ for the assumed 1966-71 time period.) Since Apollo missions are restricted to $-45° \leq \lambda_s \leq 45°$, and recalling that $7° \leq E_{sm} \leq 20°$, use of Equation (15) will show that E_o is restricted to the range $-38° \leq E_o \leq 65°$. This region is indicated by the shaded area of Figure 7. It represents approximately 29% of the total area of the figure. The lunar lighting constraint then reduces the number of launch opportunities to 29% of the total, or about 8 days per month provided that any launch site within the above mentioned limits on λ_s can be freely selected.

(3) Earth Launch Lighting

In this section the effect of restricting launches to the daylight hours will be developed assuming that there are no lunar lighting constraints. The lighting conditions at earth launch are determined by the sequence of equations given below:

Equations	Inputs	Outputs
(1), (2)	i_s, QA_s	RA_s, DEC_s
(4), (5)	i_m, QA_m, RA_{node}	DEC_m, RA_m
(6)	AL, RA_m, DEC_m, ϕ_L	RAL_A, RAL_P
(9), (10), (11)	RA_s, RAL_A, RAL_P	E_{seA}, E_{seP}

In the above, QA_s and QA_m are the independent variables, i_m and AL are parameters which assume a set of fixed values, i_s and ϕ_L are constants, and E_{seA} and E_{seP}, the elevation of the sun for the Atlantic and Pacific launches, are the output variables. Figures 8-15 are contour graphs for the conditions for sunrise and sunset (that is, for $E_{seA} = E_{seP} = 0$) for the launch azimuths of 72°, 90°, and 108°. The daylight and night hours are represented by the area between the curves. The area representing lunar lighting conditions is also shown. Note that the earth launch curves are much more complicated than the lunar landing curves. This is caused by the complex behavior of the inertial position vector of the launch site. Data used for the figures is given in the table below:

Figure	Dates	i_m	RA_{node}
8-9	1966-71	28.34°	0°
10-11	1971-76	23.45°	-13°
12-13	1976-80	18.30°	0°
14-15	1980-85	23.45°	+13°

If the locus of the moon's motion were drawn in as was done in Figure 6, then the length of the line segment in the daylight area will be proportional to the number of days per month that a launch occurs in daylight. This was done with Figures 8 and 12, the entire year being covered by lines corresponding to the 13-odd lunar months that make up one year. The length of the line segment that fell in the daylight area was further subdivided into in-plane and out-of-plane parts using the criteria that in-plane Pacific injection launches occur in the interval $90° \leq QA_m \leq 270°$, as shown in Section II. The results are given in Figure 16. It can be seen that as pointed out in Reference (1) all in-plane daylight launches occur in spring and summer, and that during most of this time the number of daylight in-plane launches greatly exceeds the number of daylight out-of-plane launches. The latter show a fairly uniform distribution throughout the year. A geometrical explanation of this behavior is given in Appendix III. The in-plane curve for $i_m = 28.3°$ exhibits a flat top which is not present in the corresponding curve for $i_m = 18.3°$, but in general the effect of a change in i_m is not great.

The Atlantic injection curves (Figures 9 and 13) are identical in shape to the corresponding Pacific curves (Figures 8 and 12) except that QA_m is 180° greater for the same value of QA_s. Figure 16 therefore applies to either Atlantic or Pacific injections.

In obtaining the results just discussed the daylight hours have been taken to be the area between the curves for which AL = 72° for sunrise and AL = 108° for sunset. The full 4-1/2 hour launch window is then available throughout the month. It can be seen from Figures 8-15 that the daylight area would be considerably increased by reducing the 108° launch azimuth limit for the sunset boundary, and increasing the 72° limit for the sunrise boundary. Such a change would, of course, result in a reduced launch window; however, Figures 8-15 indicate how the window should be truncated to keep the reduced window entirely in daylight.

(4) Lunar Lighting Constraints with Daylight Launches

In this section the effect of satisfying both earth launch lighting and lunar lighting constraints will be worked out. The area which represents acceptable lunar lighting which was shown in Figure 7 has also been indicated in Figures 8-15. The common area gives the condition under which both lighting constraints are satisfied. It is immediately evident from Figures 9, 11, 13 and 15 that except for a small region in Figure 9 there is no common area, and all Atlantic injection launches are eliminated. On the other hand, as shown in Figures 8, 10, 12 and 14, there are many Pacific injection launches which satisfy both lighting constraints. This behavior can be understood by remembering that the lunar lighting conditions are approximately satisfied when the sun is 90° behind the moon referenced to an earth centered coordinate system. The Pacific launch vector is also behind the moon, and will therefore be located in daylight. Similar considerations will show that the Atlantic launch vector will in general be in darkness. (See Appendix III.) From Figure 8 it can be seen that the most favorable launch times occur when QA_s is in the range $45° < QA_s < 150°$, when the acceptable area is limited only by the lunar lighting constraints. This corresponds to the time period from about May to August. The least favorable launch time is in the vicinity of $QA_s = 0$ (March 21). The common area of Figure 12 ($i_m = 18.3°$) is somewhat larger than that of Figure 8, ($i_m = 28.3°$) but in general the effect of a change in i_m is not great.

In Figure 8, the line $E_o = 0$ is tangent to the boundary of the common area at $QA_s = 0$ and 180 degrees. Note that along this line both lighting restrictions are satisfied throughout the year. From Equation (15) $E_{sm} = \lambda_s$ along this line, and consequently λ_s should lie within the range $7° < \lambda_s < 20°$ to satisfy both lighting restrictions throughout the year. Expressed another way, the lunar landing sites should be biased in the positive direction by 13-1/2° (the center of the above range in λ_s) for the best average lighting conditions at both earth launch and lunar landing.

To increase the amount of daylight after launch in case of a downrange abort, the sunset curves of Figures 8-15 could have been specified by setting E_{se} to, say, 15° instead of zero. This will result in approximately one hour of additional daylight. As shown by Equation 13 the sunset curves will be shifted to the right by about 15°, reducing the common area somewhat.

In the previous analysis it has been assumed that earth launch and lunar lighting are of equal importance and moreover that daylight should occur during the entire launch window. It is probably more realistic to adopt the point of view that the lighting at lunar landing is more important, and that daylight launches are desirable, but not necessary. Atlantic injection launches will then become available, and Figures 8-15 can be used to determine the approximate time of day that launch will take place.

VI. CONCLUSIONS

To a good approximation the elevation of the sun (E_{sm}) at a lunar landing site is governed by linear equations which appear as straight lines when plotted with the angular position of the sun (QA_s) and the moon (QA_m) as the coordinate variables. When E_{sm} is restricted to the range $7° < E_{sm} < 20°$, there is essentially only one launch opportunity per month for a single landing site. When in addition the landing site longitude λ_s is given the range $-45° < \lambda_s < +45°$, the number of launch opportunities rises to about 8 per month. At a fixed time E_{sm} and λ_s are related by the equation $E_{sm} - \lambda_s = \text{constant}$.

In contrast the elevation of the sun at earth launch is a complex function which must be represented by a family of curves in the $QA_s - QA_m$ coordinate system mentioned above. The curves which represent sunrise and sunset for Atlantic and Pacific injection launches are identical in shape, and are displaced from each other by 1/2 lunar month. It is shown in the text that in-plane daylight launches occur only in the spring and summer and out-of-plane launches occur throughout the year.

Nearly all Atlantic injection launches are eliminated if both the lunar lighting restriction and daylight launches for the full launch window are required. On the other hand there are many Pacific injection launches which satisfy both restrictions. For these launches the best launch period is from May to August.

The high correlation between daylight launches and Pacific injections (with the current lunar lighting constraint) is apparent when the position of the earth launch site is geometrically related to the position of the target earth-moon vector. It is shown that the right ascension of the launch site for Pacific injections <u>trails</u> this earth-moon vector by 0° - 180° over a lunar cycle while the right ascension of the launch site for Atlantic injections <u>leads</u> by 0° - 180°. Under the current lunar lighting constraint the sun <u>trails</u> the earth-moon vector by about 90°. When expressed in this manner, it is apparent that a very high percentage of the daylight launches are associated with Pacific injections.

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L. P. Gieseler

Attachments
References
Appendixes I, II and III
Figures 1 through 21

BELLCOMM, INC.

REFERENCE

1. V. S. Mummert, "Earth Launch Lighting," Bellcomm Memorandum for File, January 27, 1964.

APPENDIX I

LIST AND DEFINITION OF SYMBOLS

- AL - launch azimuth
- DEC_m - declination of the moon
- DL - dihedral angle (general)
- DL_A - dihedral angle (Atlantic injection)
- DL_P - dihedral angle (Pacific injection)
- D_L - length of the longest day at Cape Kennedy
- D_s - length of the shortest day at Cape Kennedy
- DEC_s - declination of the sun
- E_{se} - elevation of the sun above the horizon at the earth launch site
- E'_{se} - approximate value of E_{se}
- E_{seA} - E_{se} for Atlantic injection launches
- E_{seP} - E_{se} for Pacific injection launches
- E_{sm} - elevation of the sun above the eastern horizon at the lunar landing site
- E_o - E_{sm} for $\lambda_s = 0$
- i_m - inclination of lunar orbit plane with respect to the equatorial plane of the earth
- i_s - angle between ecliptic and equatorial planes ($\cong 23.45°$)
- QA_m - angular position of moon in its orbit, measured from its ascending node
- QA'_m - angular position of the moon as defined in Figure 3
- QA_s - angular position of sun measured in the ecliptic from the vernal equinox

Appendix I

RAL_A - right ascension of launch site for Atlantic injection launches

RAL_P - right ascension of launch site for Pacific injection launches

RA_{node} - right ascension of moon's ascending node, the nodal line being the intersection between the lunar orbital plane and the equatorial plane

R_m - side of spherical triangle shown in Figure 1b

RA_m - right ascension of the moon

RA_s - right ascension of the sun

w_e - angular velocity of the earth relative to the sun ($= 15°/hr$)

λ_s - selenographic longitude of landing site

ϕ_L - launch site latitude ($\cong 28.5°$ geocentric)

APPENDIX II

DETERMINATION OF LAUNCH SITE VECTOR

Figure 17a is a view of the northern hemisphere of the earth, looking directly at the meridian which passes through the earth-moon vector. The spacecraft orbit is shown passing through the launch site, making an angle AL with the launch site meridian. Applying spherical trigonometry to the spherical triangle whose angles are C_A, B_m, and 180 - AL, we obtain:

$$B_m = \sin^{-1}\left(\frac{\sin AL \cos \phi_L}{\cos(DEC_m)}\right) \quad 0 < B_m < 90° \qquad (17)$$

$$C_A = 2\tan^{-1}\left(\frac{\cos 1/2\ (\phi_L - DEC_m)}{\cos 1/2(180-DEC_m-\phi_L)\tan 1/2(180-AL+B_m)}\right) \qquad (18)$$

$$0 \leq C_A \leq 180°$$

$$RAL_A = RA_m + C_A \qquad (19)$$

where RAL_A is the right ascension of the launch site which is ahead of the earth-moon vector. A launch from this site is identified as an Atlantic injection launch since the translunar injection point is in most cases east of Cape Kennedy.

Figure 17b illustrates the second solution for the location of the launch site vector. Here the launch site is behind the earth-moon vector. A launch from this site is known as a Pacific injection launch since the translunar injection point is in most cases west of Cape Kennedy. The equation for C_P becomes

$$C_P = 2\tan^{-1}\left(\frac{\cos 1/2\ (\phi_L - DEC_m)}{\cos 1/2(180 - DEC_m - \phi_L)\tan 1/2(AL + B_m)}\right) \qquad (20)$$

$$0 \leq C_P \leq 180°$$

$$RAL_P = RA_m - C_P \tag{21}$$

where RAL_P is the right ascension of the launch site shown in Figure 17b. By comparing Equations (18) and (20) it can be seen that C_A equals C_P when the respective values of AL are supplementary.

The dihedral angle DL is the angle between the spacecraft orbit and the lunar orbit planes, as shown in Figure 1b. It is also shown in Figure 17a, where it is designated by DL_A, and in Figure 17b, where it appears as DL_P. From the figures it can be seen that

$$DL_A = B_m - F_m$$

$$DL_P = 180 - B_m - F_m$$

where B_m can be computed by Equation (17) and F_m can be computed by the equation

$$F_m = \cos^{-1} (\cos R_m \sin i_m)$$

The dihedral angle does not appear in any of the lighting formulas, but is used as an output variable to distinguish between in-plane and out-of-plane launches.

The behavior of C_A, C_P, RAL_A, RAL_P, DL_A and DL_P are illustrated in Figures 18-21. QA_m was varied from 0 to 360°, AL was set equal to 72°, 90°, and 108°, and i_m was set equal to 18° and 28°. Figure 18 shows how C_A and C_P vary as a function of QA_m. The curves have a positive slope in the interval of $90° < QA_m < 270°$, and a negative or zero slope elsewhere. The curves of Figure 20 (RAL_A and RAL_P) show a nearly horizontal portion joined to a portion having an appreciable positive slope. The curves of Figure 19 are similar, but with a smaller difference in slope between the two portions. The general trend can be explained as follows: RA_m is always nearly equal to QA_m and therefore can be represented approximately by a straight line starting at the origin having a slope of unity. (This is shown as line B-B in Figures 19 and 20.) By Equation 19 the curve for RAL_A can be constructed by adding together corresponding ordinates from the C_A and RA_m curves. There will be a steep portion in the interval $90° < QA_m < 270°$ where both slopes are positive, and a relatively flat portion in the remainder of the interval where the slope of the C_A curve is negative. The behavior of the RAL_P curve can be similarily explained using Equation (21).

The behavior of DL_A and DL_P is shown in Figure 21 for i_m = 18 and 28 degrees. The values of QA_m for in-plane and out-of-plane launches are indicated in the drawing.

APPENDIX III

GEOMETRICAL INTERPRETATION

(1) Behavior of the Dihedral Angles

Referring to Figures 17a and 17b, during the lunar month the earth-moon vector moves along the curve representing the lunar orbit, making a complete circuit around the earth in 27.32 days. The launch site vector moves with the earth-moon vector along a line of constant latitude and for Atlantic injection launches stays ahead of it by C_A degrees. It is assumed that the declination of the moon is less than the latitude of Cape Kennedy. (This is almost always the case.) Then for Atlantic injection launches, the spacecraft orbit curve will have a positive slope at the earth-moon vector. By the same reasoning, for Pacific injection launches, the spacecraft orbit curve will have a negative slope at the earth-moon vector. It can be seen that DL_P will be less than DL_A in that part of the lunar cycle when the slope of the lunar orbit curve is negative, that is, when $90° < QA_m < 270°$. This is then the region for which Pacific launches are of the in-plane type, confirming the definitions given in Figure 21.

(2) Explanation of Figure 16

It has been indicated above that as the moon revolves once around the earth, the inertial position of the two launch vectors also makes one revolution around the earth. The behavior is quite different if the two launch vectors are designated as in-plane and out-of-plane, rather than Atlantic and Pacific injection vectors. As shown in Figure 19 and 20, as QA_m varies from 0 to 360° the in-plane launch vector remains relatively fixed in space, having an average right ascension of about 90°. From Figure 5, it can be seen that daylight will occur at a launch site location specified by RAL = 90° only if RA_s is in the range $0 < RA_s < 180°$. This confirms one of the conclusions drawn from Figure 16, that daylight in-plane launches occur only in the spring and summer months.

Reference to Figures 19 and 20 will also show that the out-of-plane launch vector makes two complete revolutions around the earth during one lunar cycle. Thus, regardless of the inertial position of the sun, there will be in general two periods during the lunar month when the launch vector is in daylight. This confirms another conclusion resulting from Figure 16, that the number of out-of-plane daylight launches per month is relatively constant throughout the year.

(3) Effect of Imposing Both Lighting Constraints

In section V it was pointed out that practically all Atlantic injection launches are eliminated when both lighting constraints are imposed, and a simple geometrical explanation of this effect was given. A different explanation can be given with the aid of Figures 19 and 20. As has been mentioned previously the lunar lighting constraint is approximately satisfied when $RA_s = RA_m - 90$. Since $RA_m \cong QA_m$, this equation can be represented by the straight line A-A indicated in Figures 19 and 20. Note that throughout the lunar month, RAL_P does not differ greatly from RA_s, and therefore, most Pacific launches will be in daylight. On the other hand, RAL_A generally differs from RA_s by about 180°, and thus, Atlantic injection launches in general will occur at night.

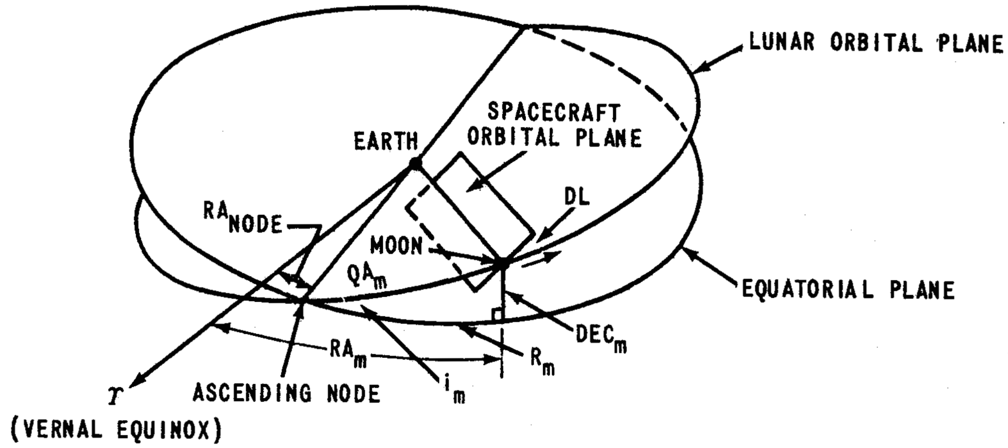

FIGURE 1b - MOTION OF THE MOON

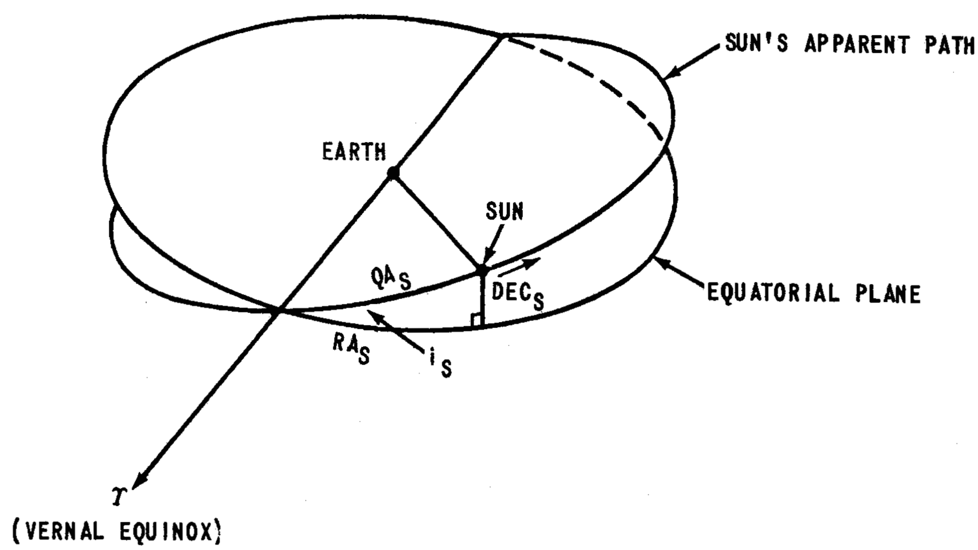

FIGURE 1a - MOTION OF THE SUN

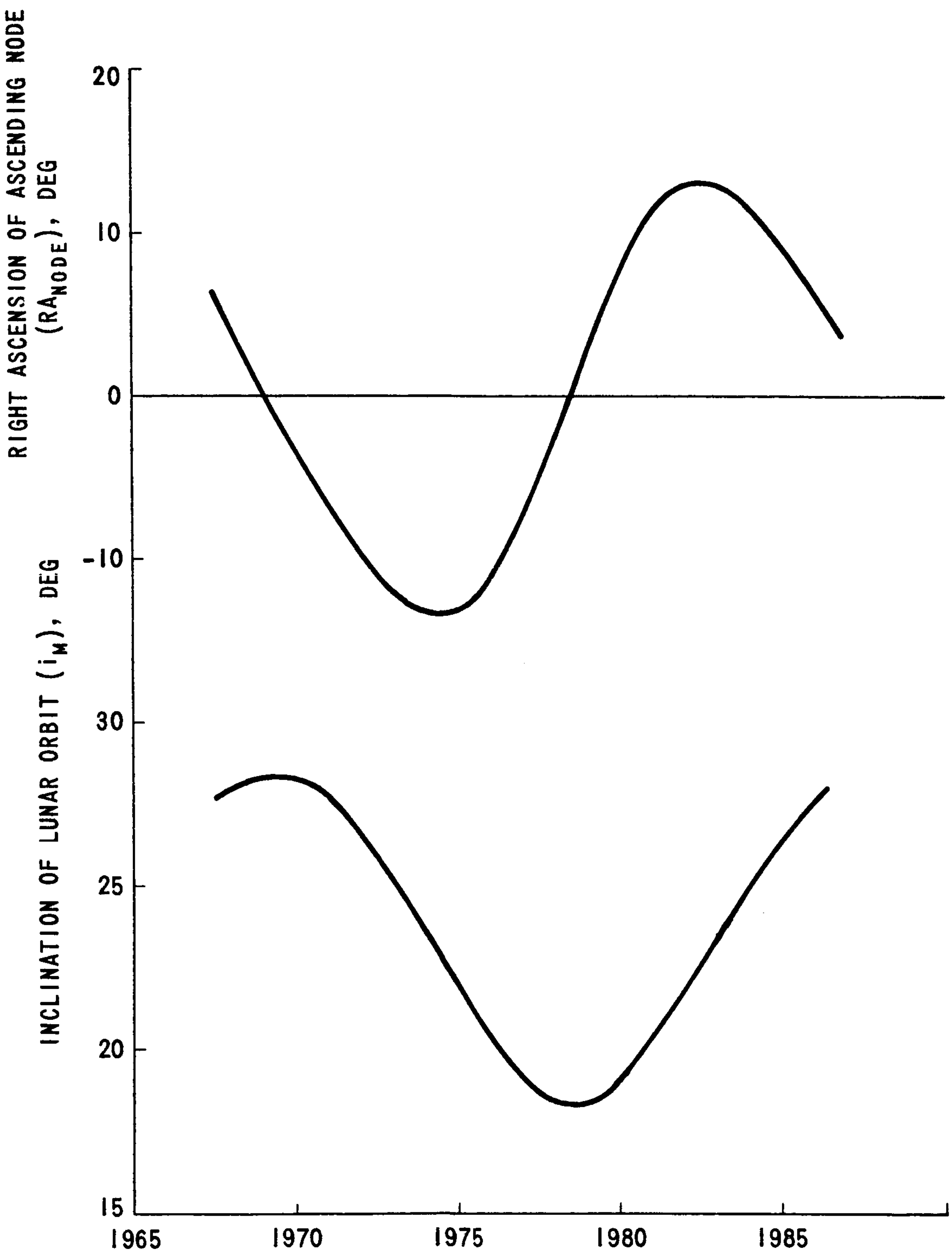

FIGURE 2 - BEHAVIOR OF LUNAR ASCENDING NODE AND INCLINATION

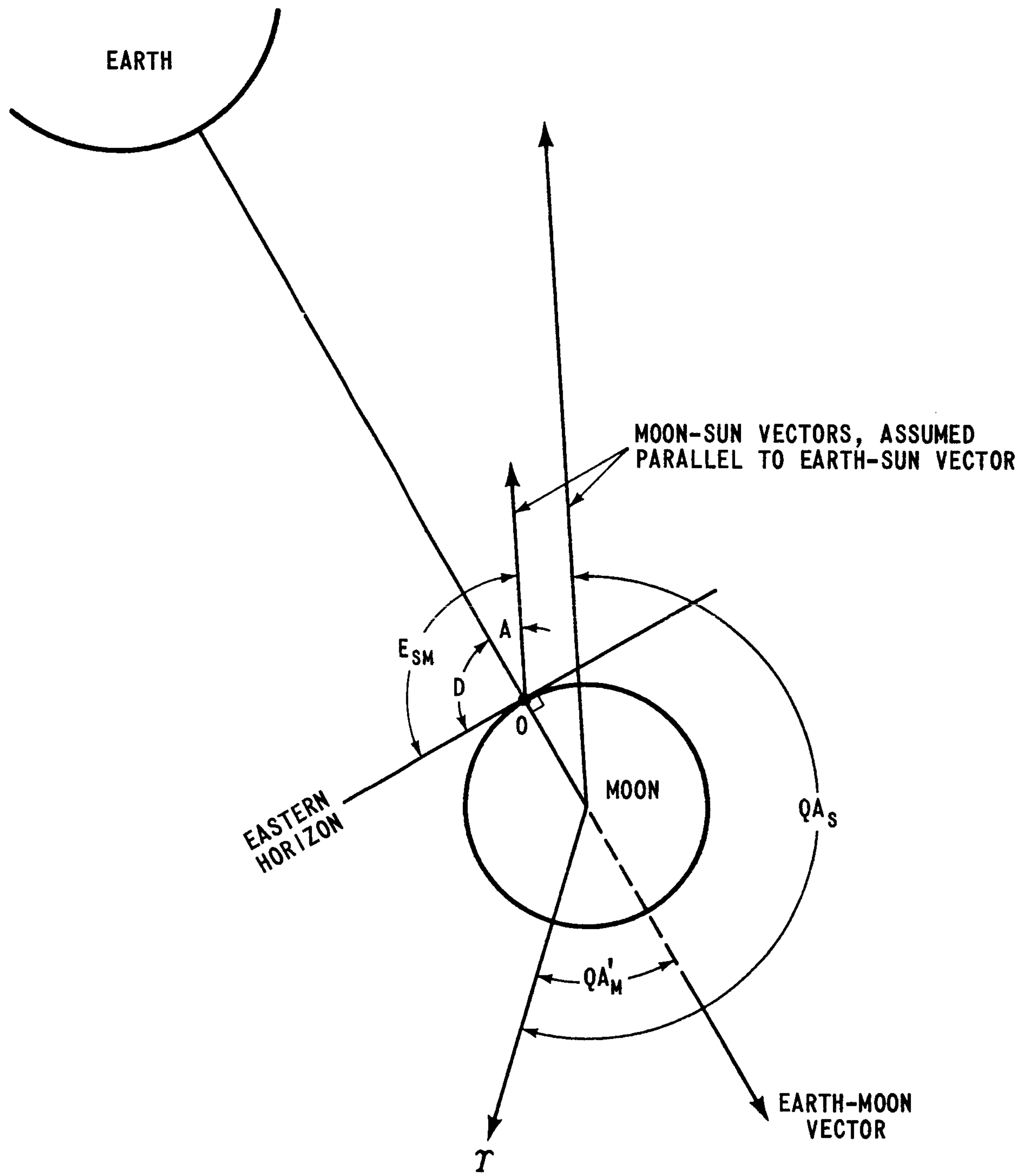

FIGURE 3 - GEOMETRY OF LUNAR LIGHTING
(SELENOGRAPHIC LONGITUDE OF OBSERVER = 0)

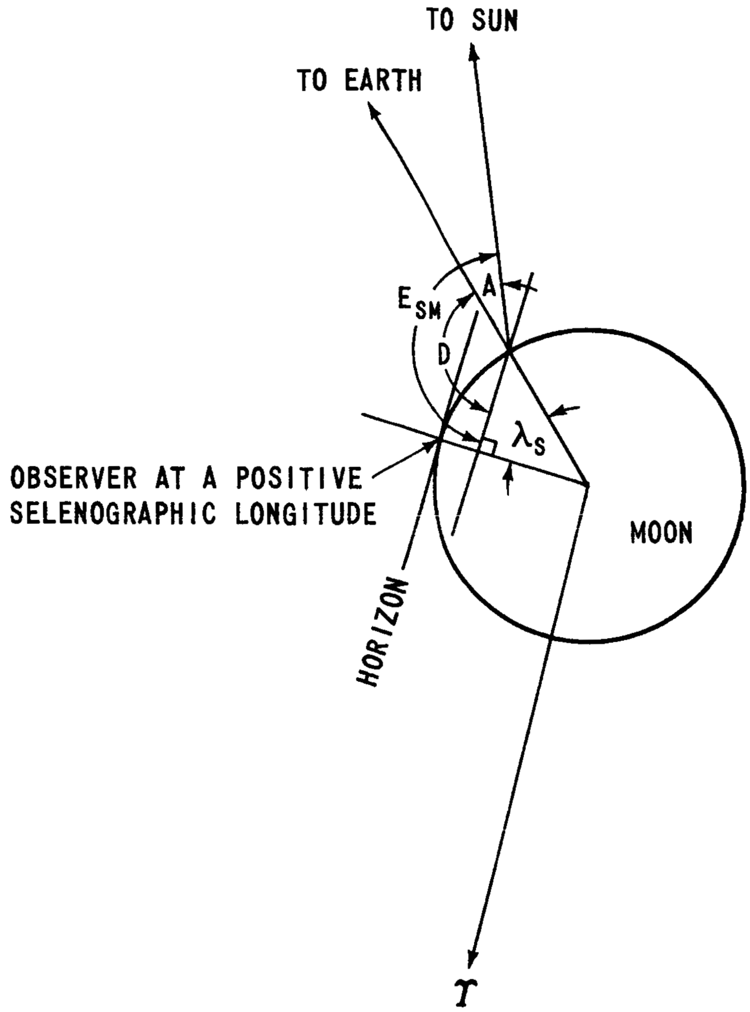

FIGURE 4 - GEOMETRY OF LUNAR LIGHTING
(SELENOGRAPHIC LONGITUDE OF OBSERVER = λ_S)

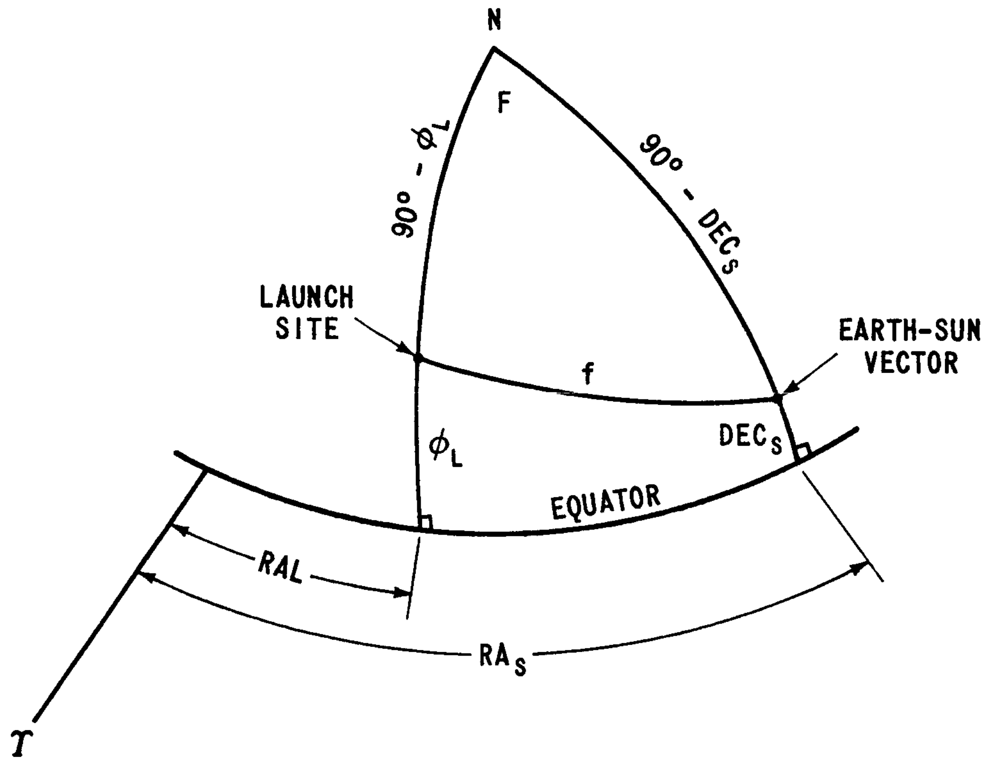

FIGURE 5 - GEOMETRY OF EARTH LAUNCH LIGHTING

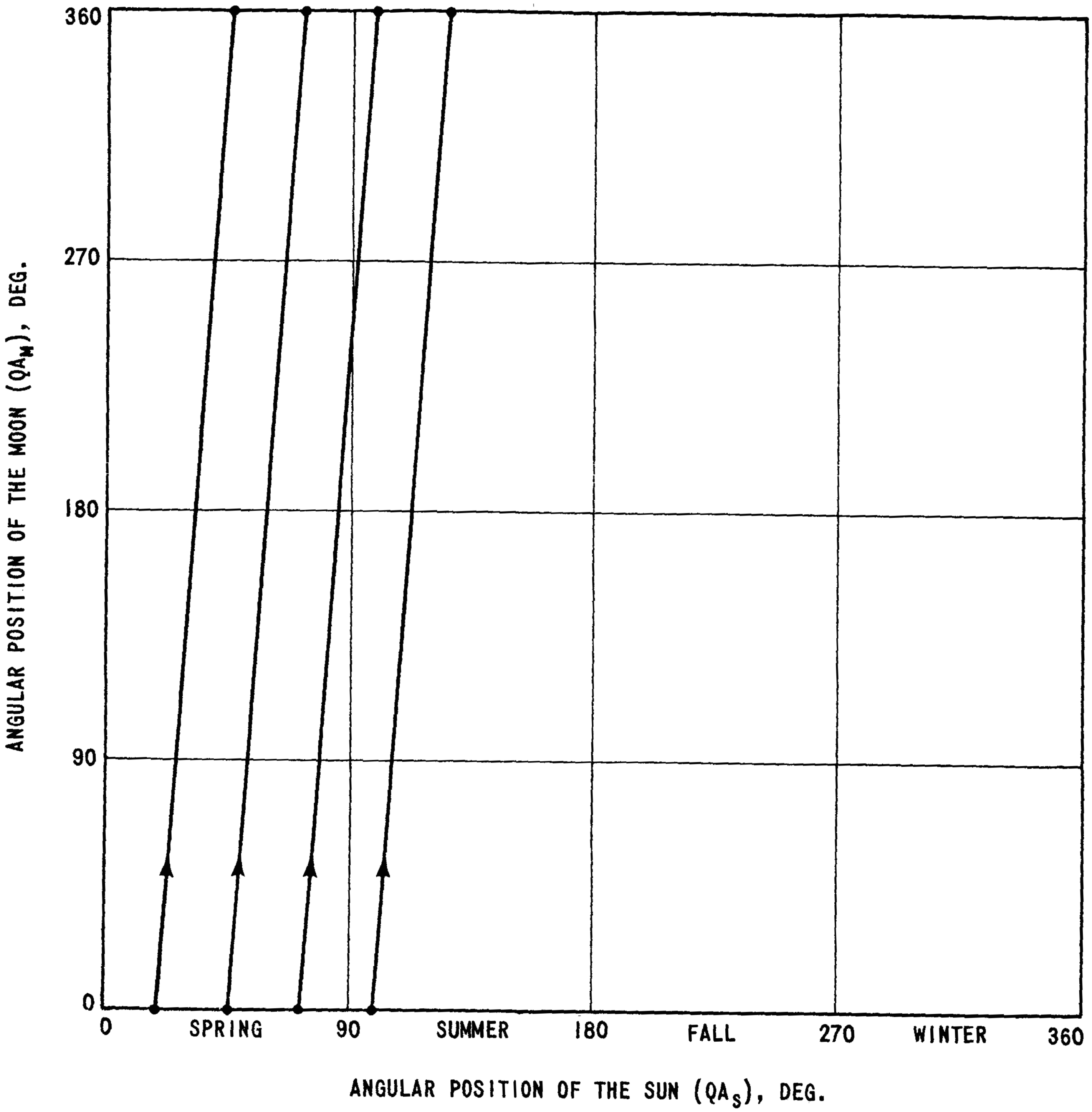

FIGURE 6 - LOCUS OF THE MOON'S MOTION

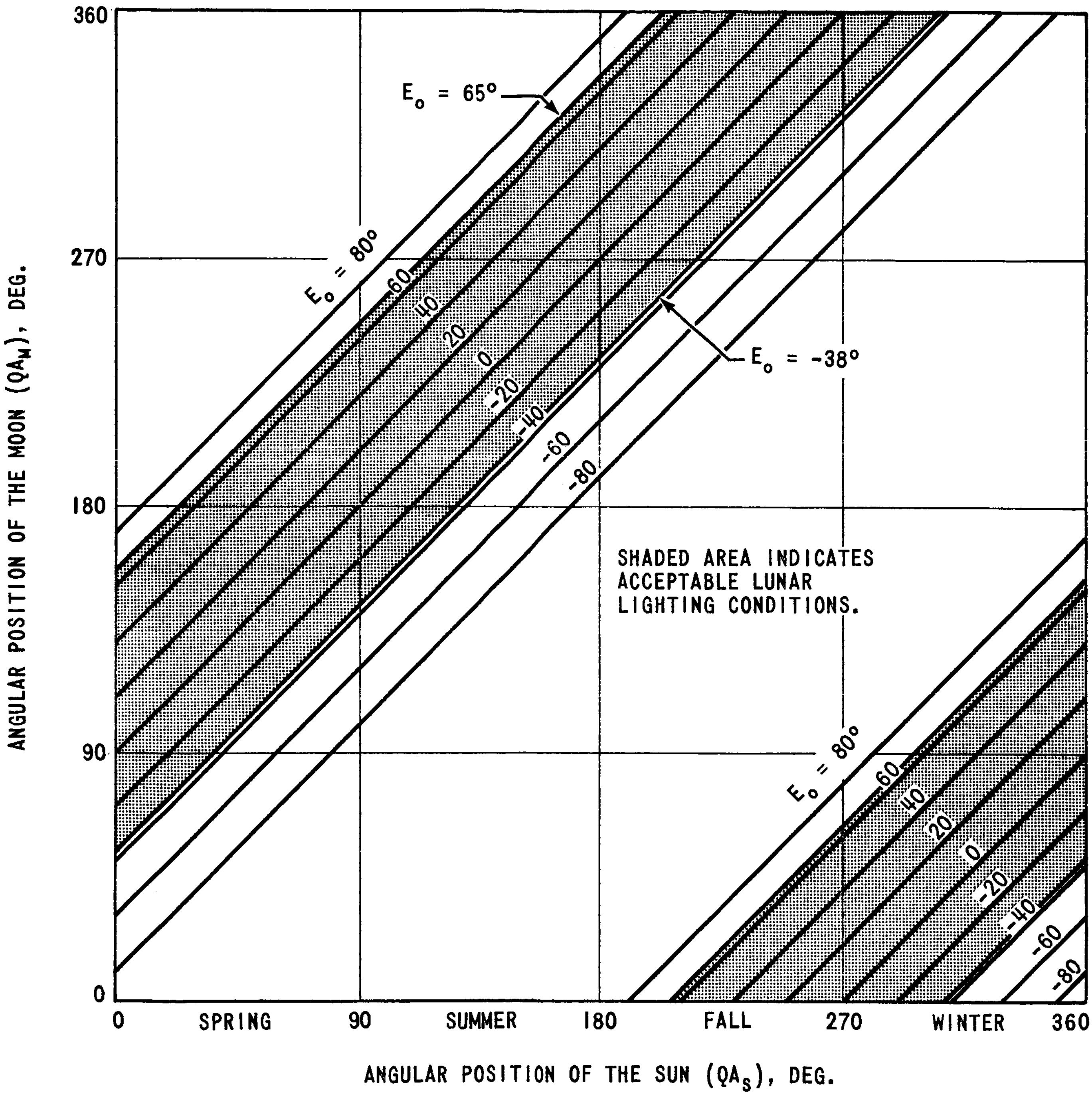

FIGURE 7 - LUNAR LIGHTING CONTOURS

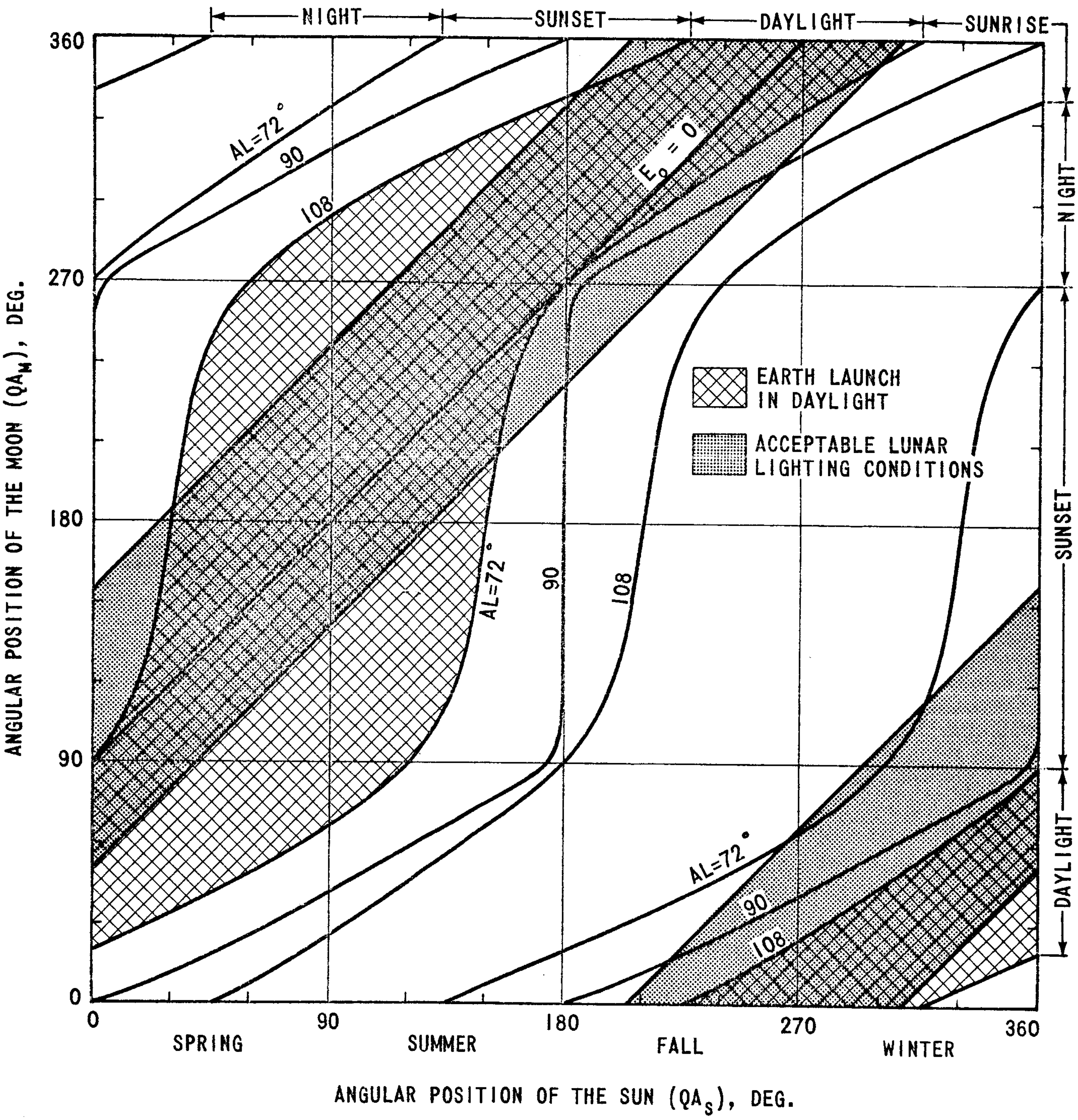

FIGURE 8 - CONTOURS FOR THE 1966 - 71 PERIOD, PACIFIC INJECTION (i_m = 28.3°, RA_{node} = 0)

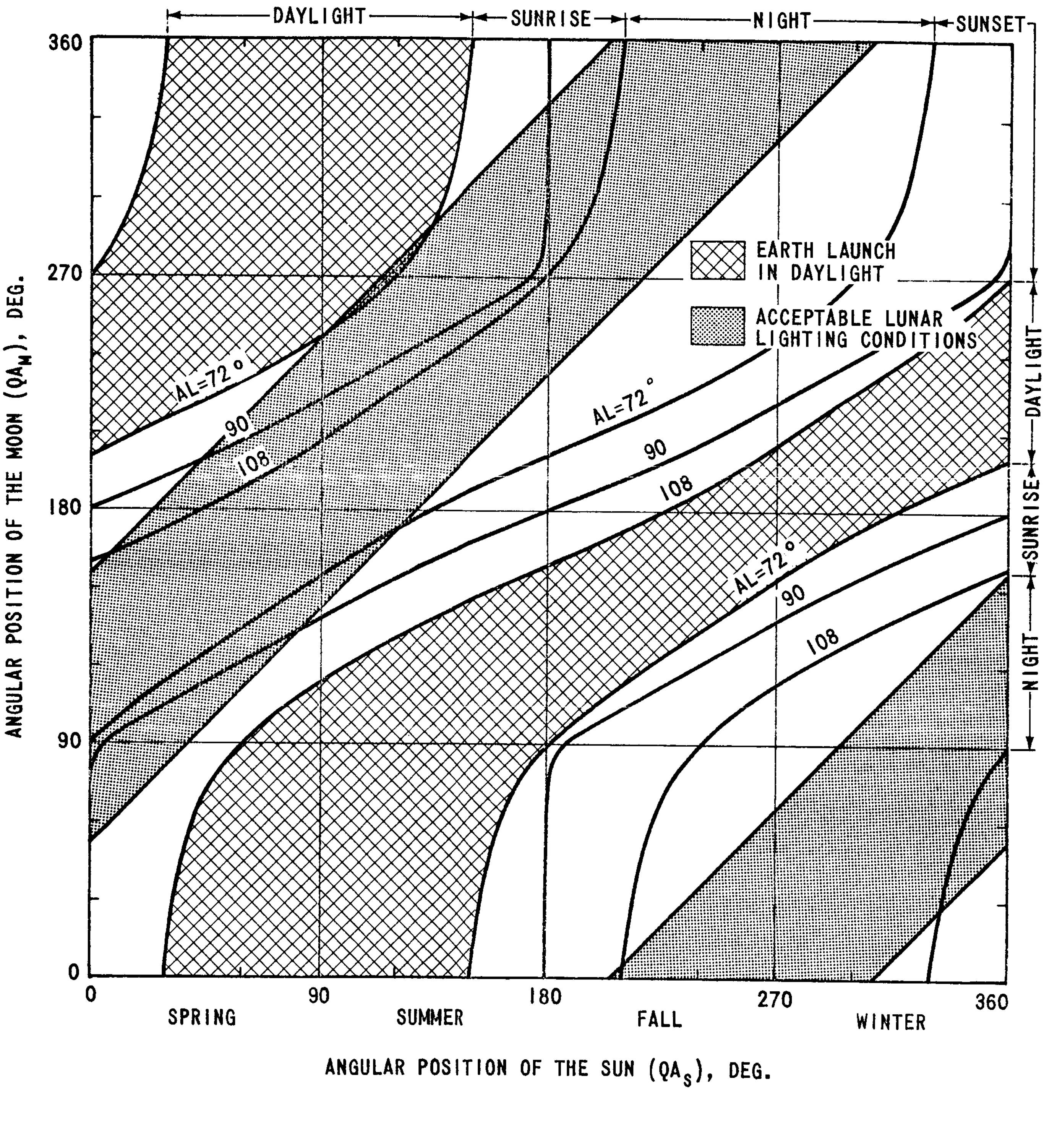

FIGURE 9 - CONTOURS FOR THE 1966 - 71 PERIOD, ATLANTIC INJECTION (i_m = 28.3°, RA_{node} = 0)

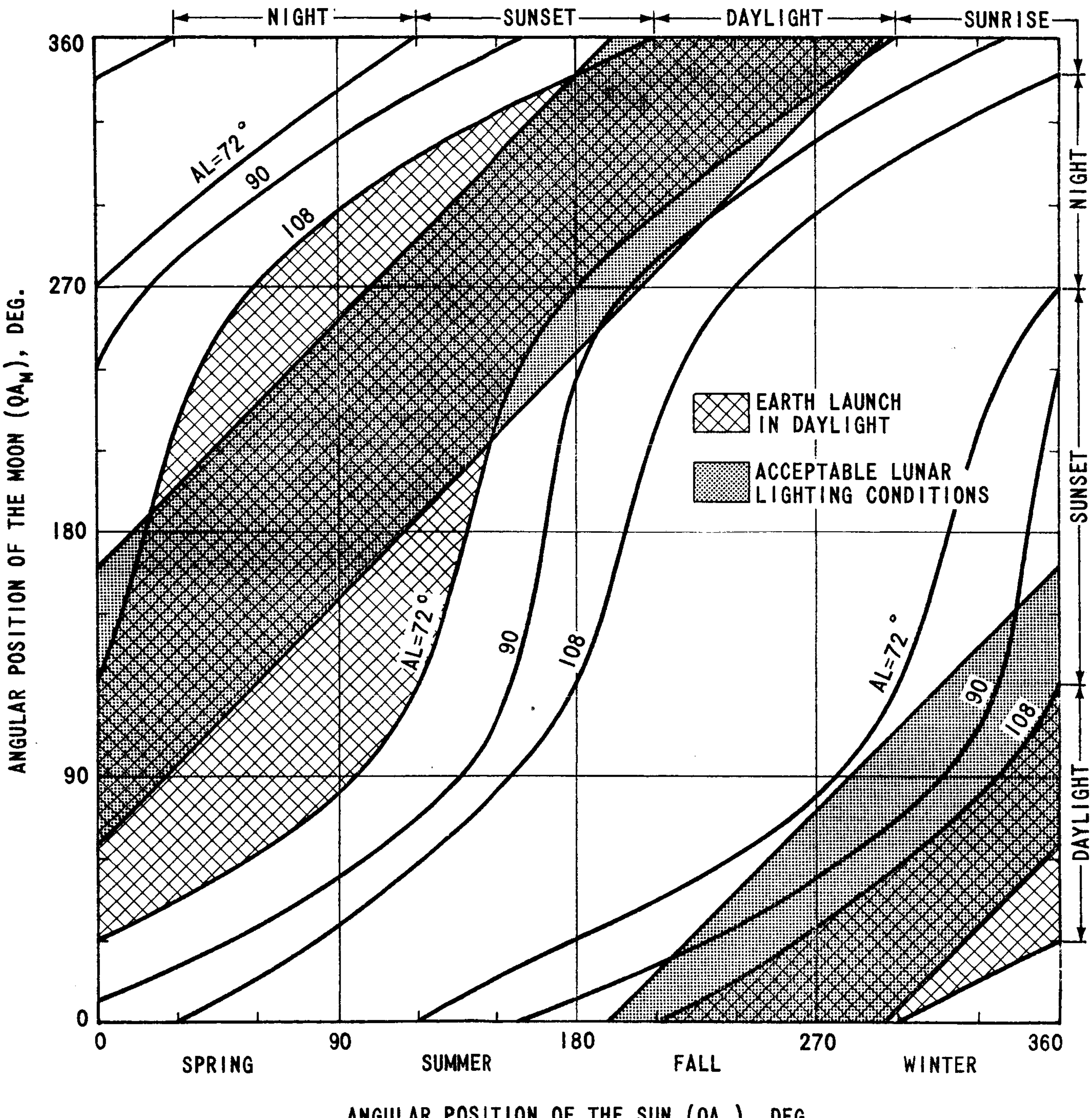

FIGURE 10 - CONTOURS FOR THE 1971 - 76 PERIOD, PACIFIC INJECTION
(i_m = 23.45°, RA_{node} = -13°)

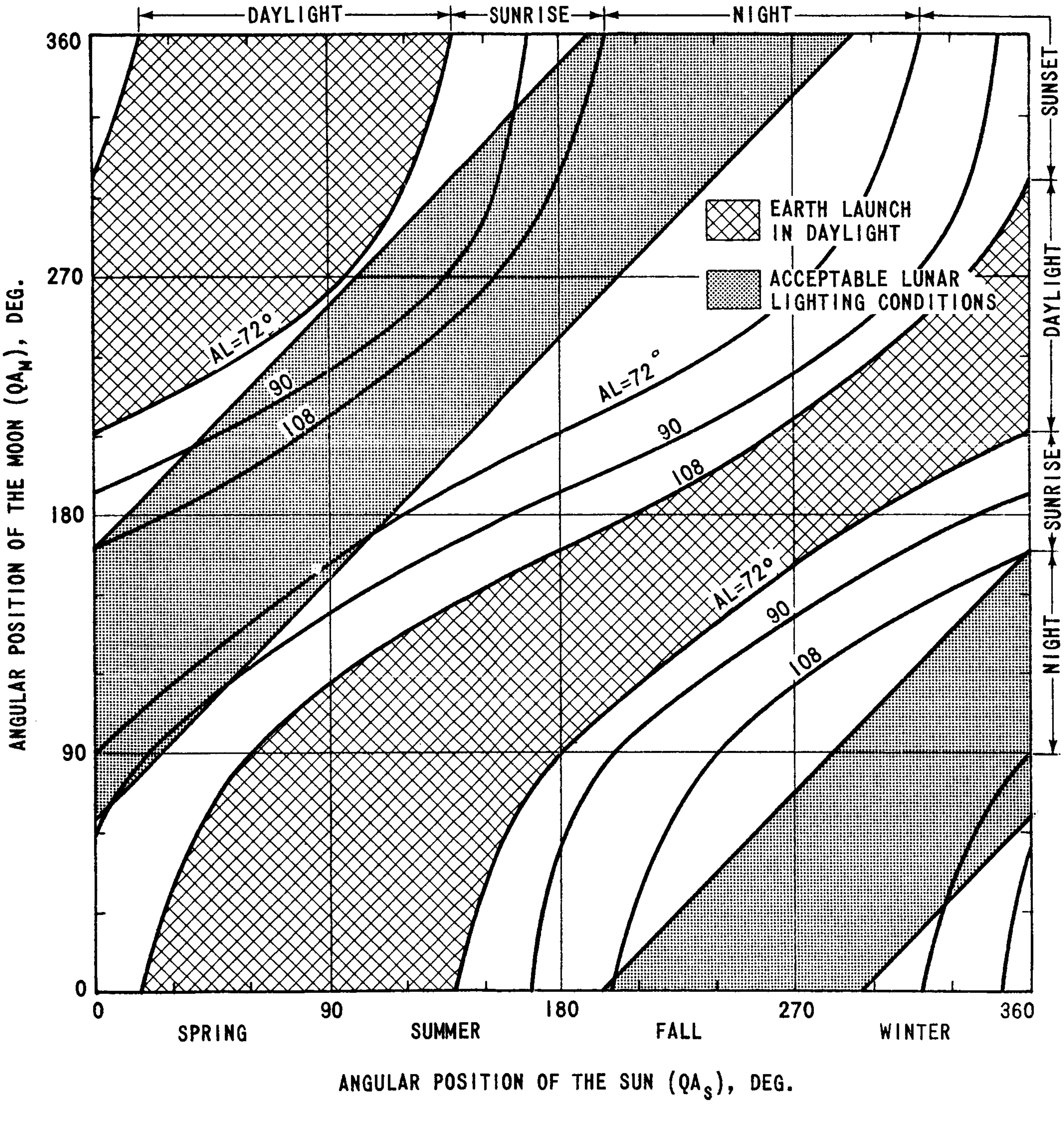

FIGURE 11 - CONTOURS FOR THE 1971 - 76 PERIOD, ATLANTIC INJECTION
(i_m = 23.45°, RA_{node} = -13°)

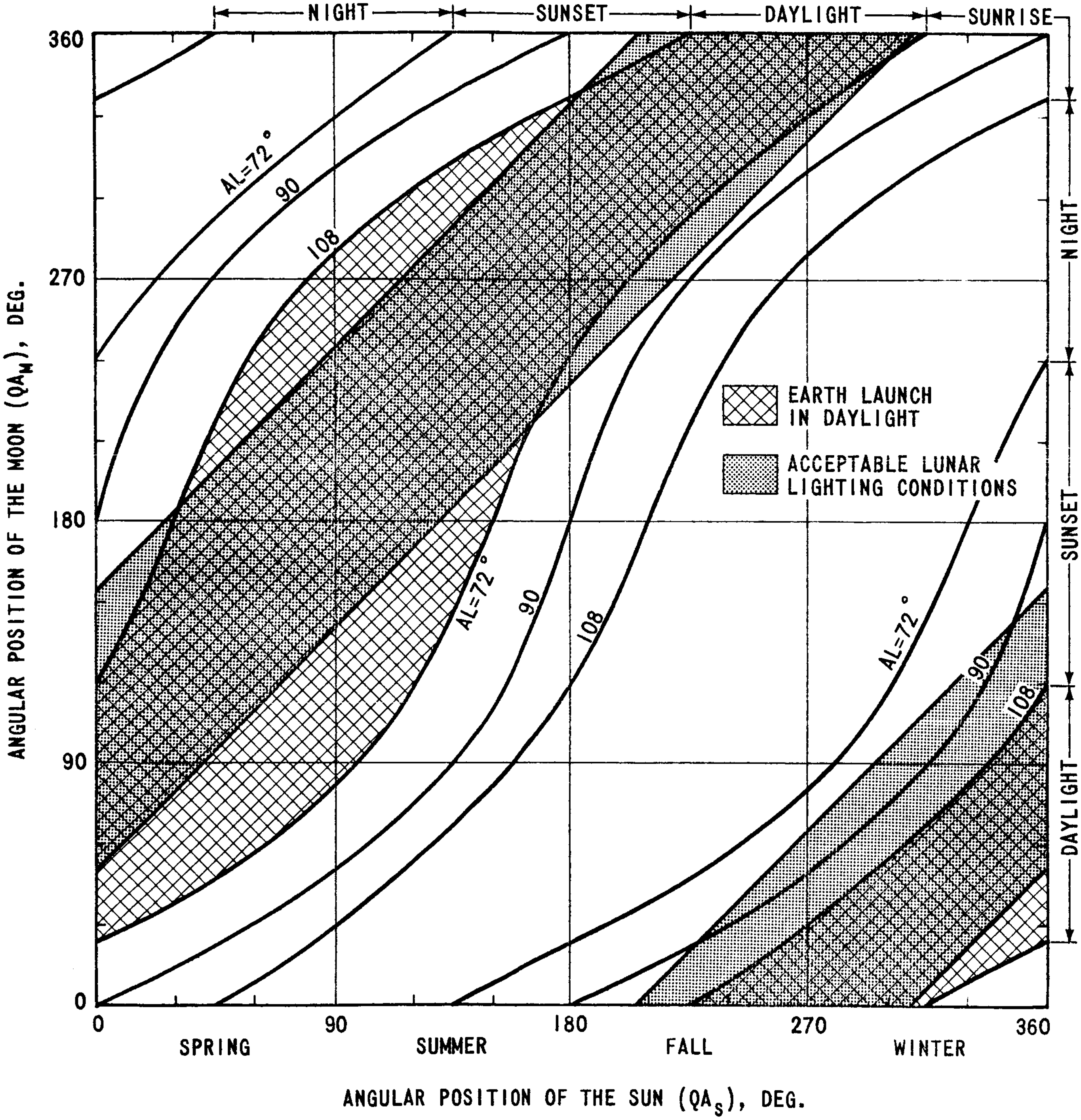

FIGURE 12 - CONTOURS FOR THE 1976 - 80 PERIOD, PACIFIC INJECTION ($i_m = 18.3°$, $RA_{node} = 0$)

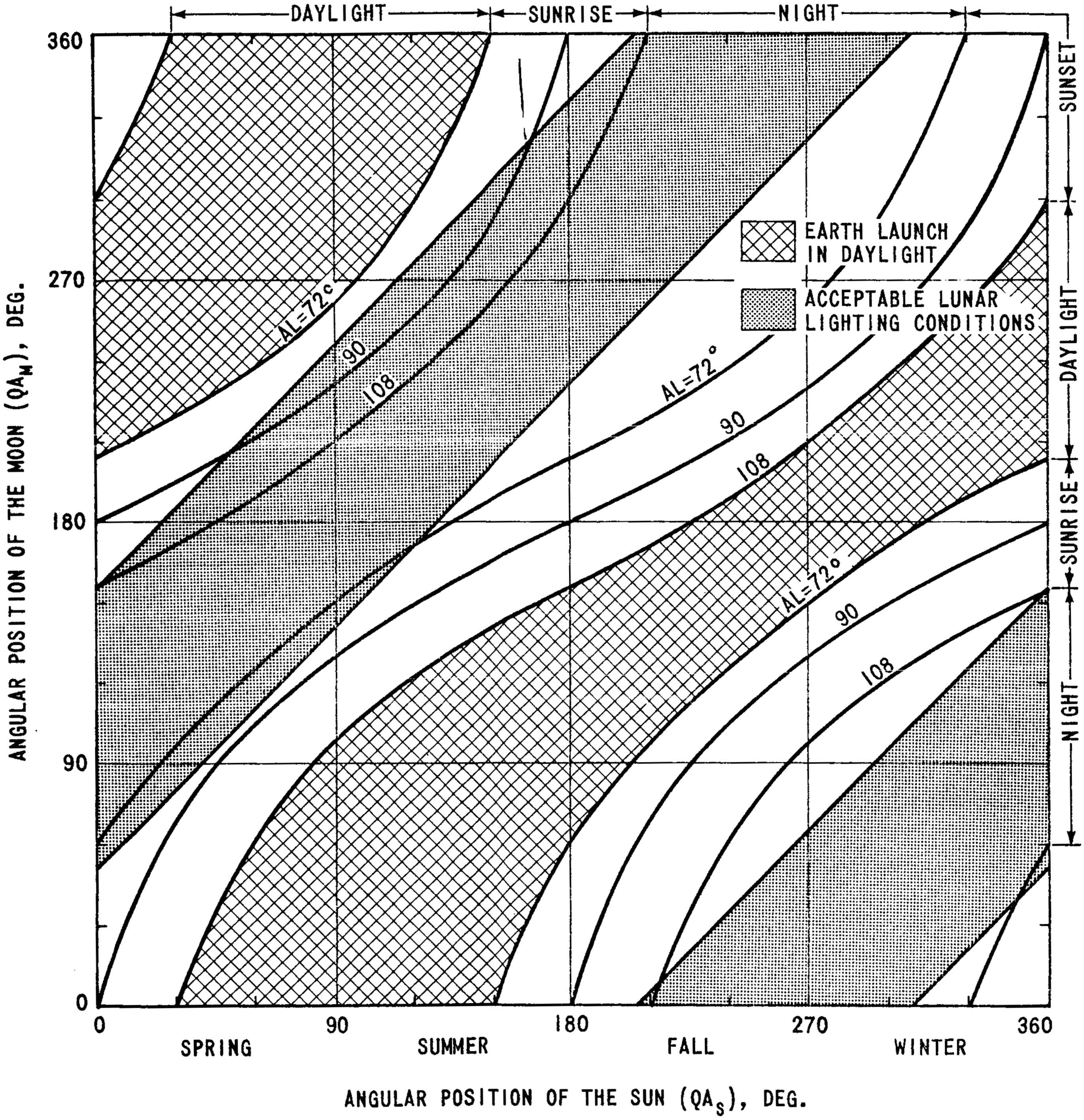

FIGURE 13 - CONTOURS FOR THE 1976 - 80 PERIOD, ATLANTIC INJECTION
($i_m = 18.3°$, $RA_{node} = 0$)

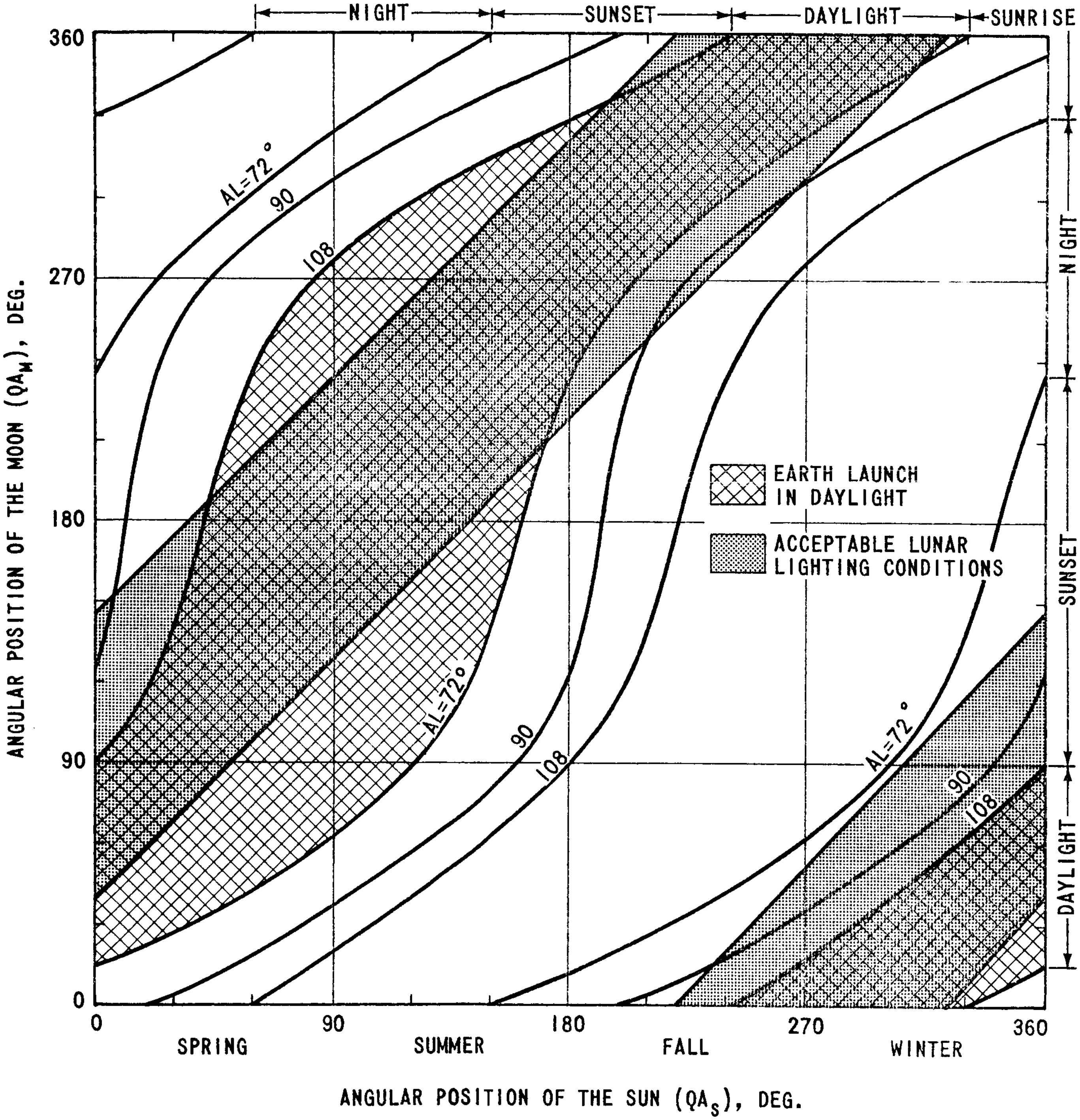

FIGURE 14 - CONTOURS FOR THE 1980 - 85 PERIOD, PACIFIC INJECTION (i_m = 23.45°, RA_{node} = 13°)

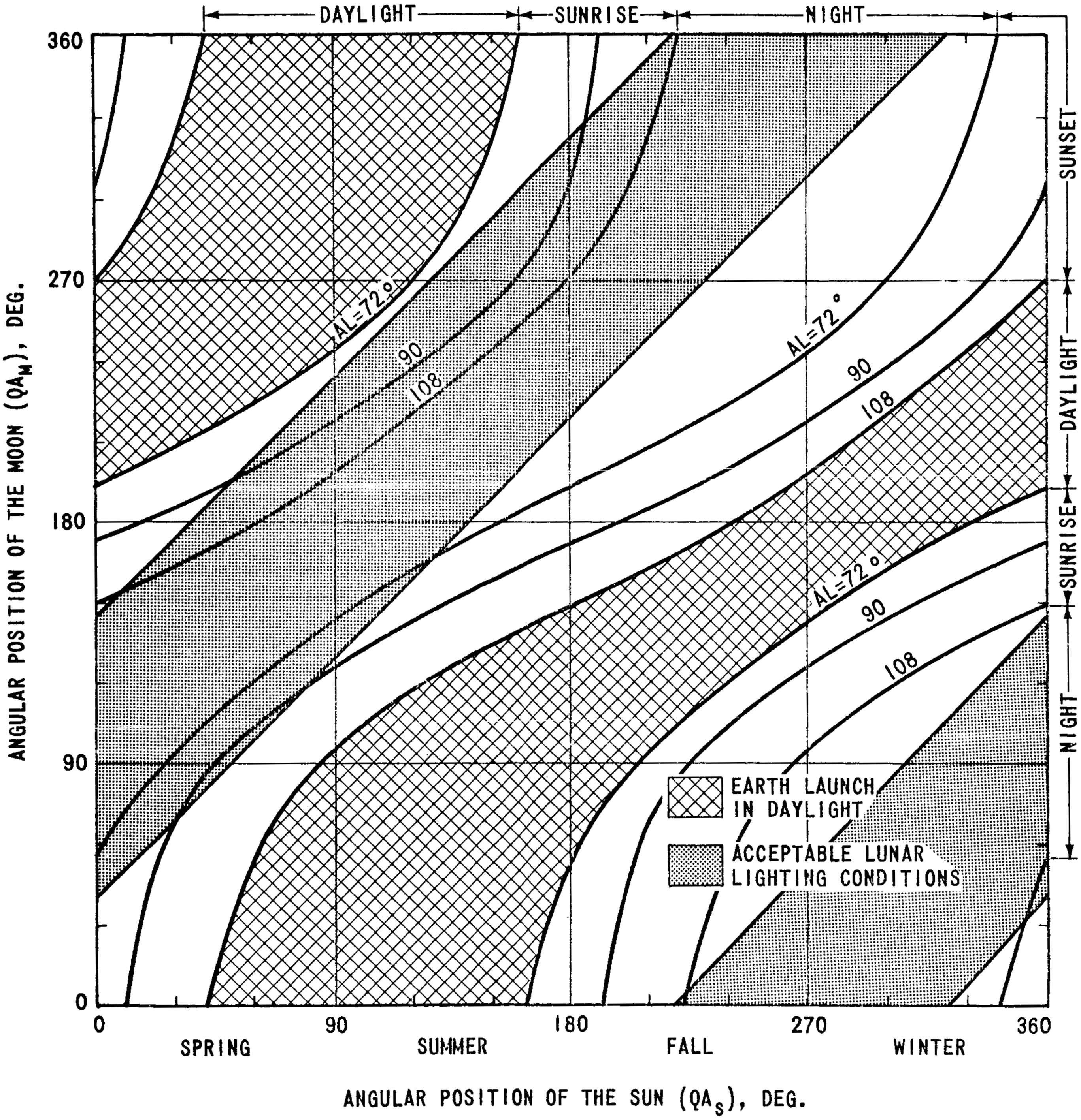

FIGURE 15 - CONTOURS FOR THE 1980 - 85 PERIOD, ATLANTIC INJECTION
(i_m = 23.45°, RA_{node} = 13°)

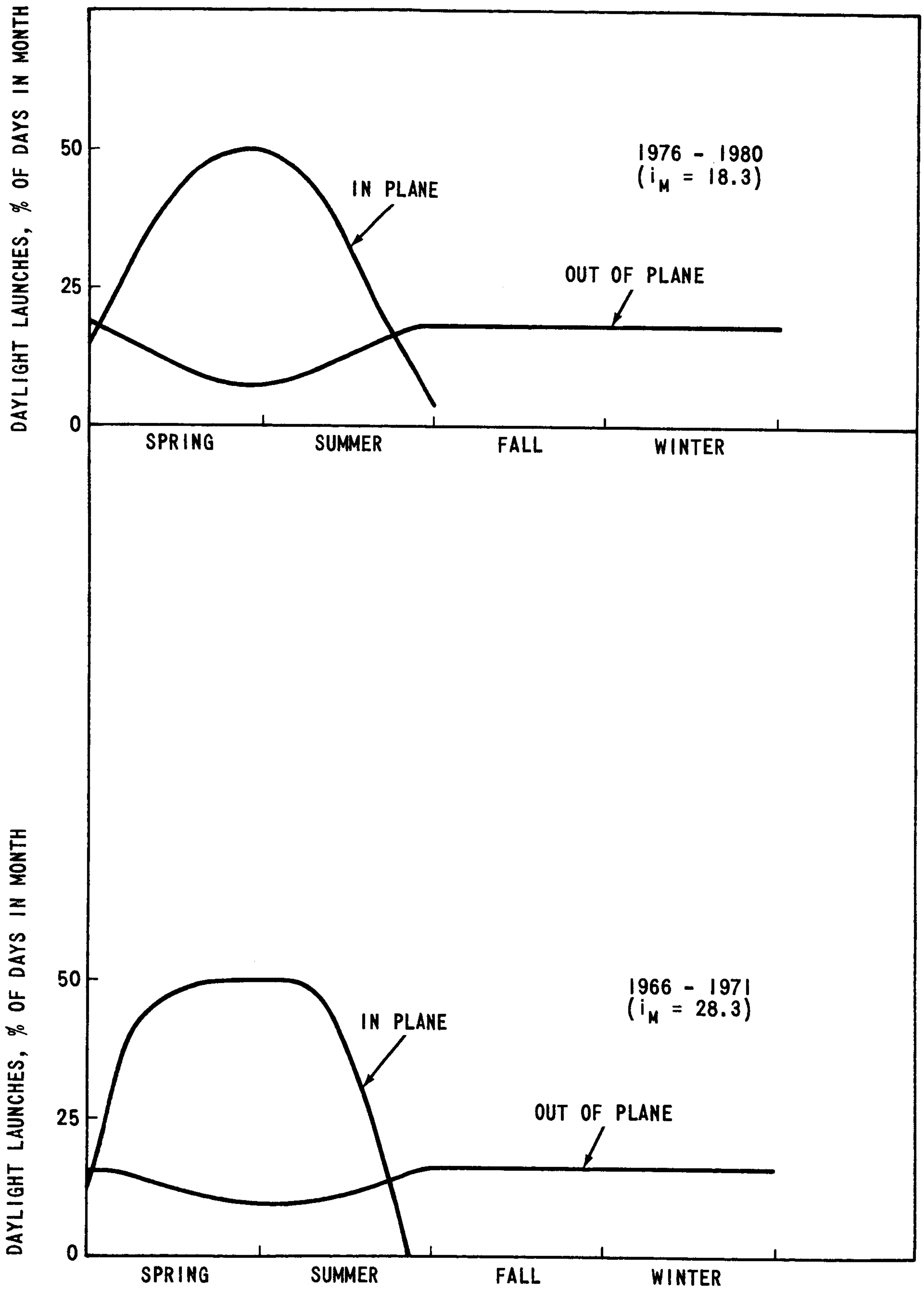

FIGURE 16 - YEARLY DISTRIBUTION OF DAYLIGHT LAUNCHES
(PACIFIC INJECTION)

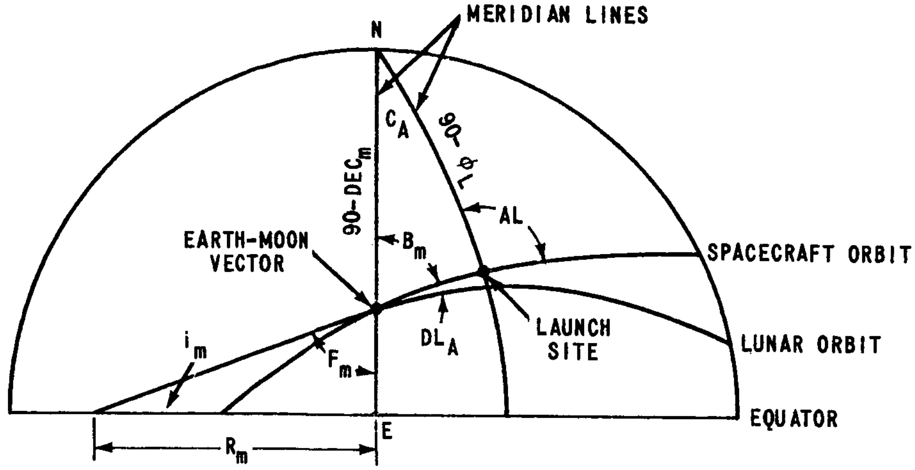

FIGURE 17a - LAUNCH SITE AHEAD OF EARTH-MOON VECTOR (ATLANTIC INJECTION)

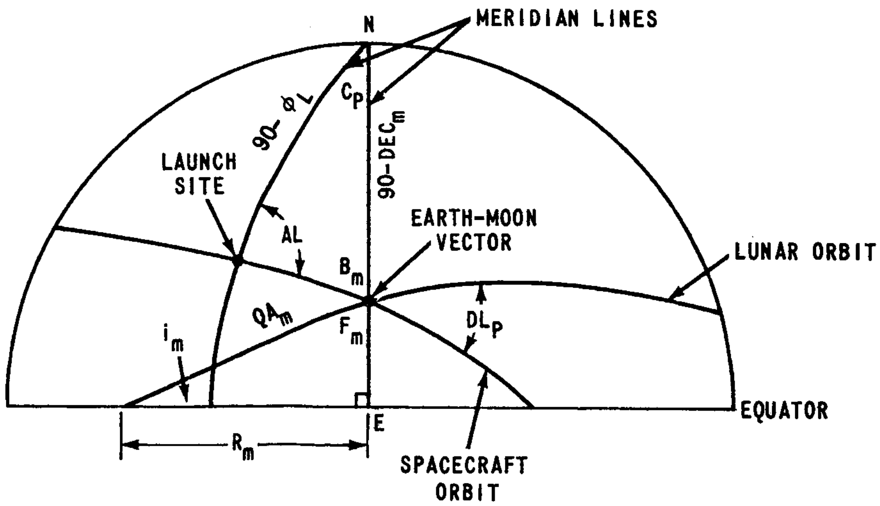

FIGURE 17b - LAUNCH SITE BEHIND EARTH-MOON VECTOR (PACIFIC INJECTION)

FIGURE 17 - SPHERICAL TRIANGLES FOR THE DETERMINATION OF THE LAUNCH SITE

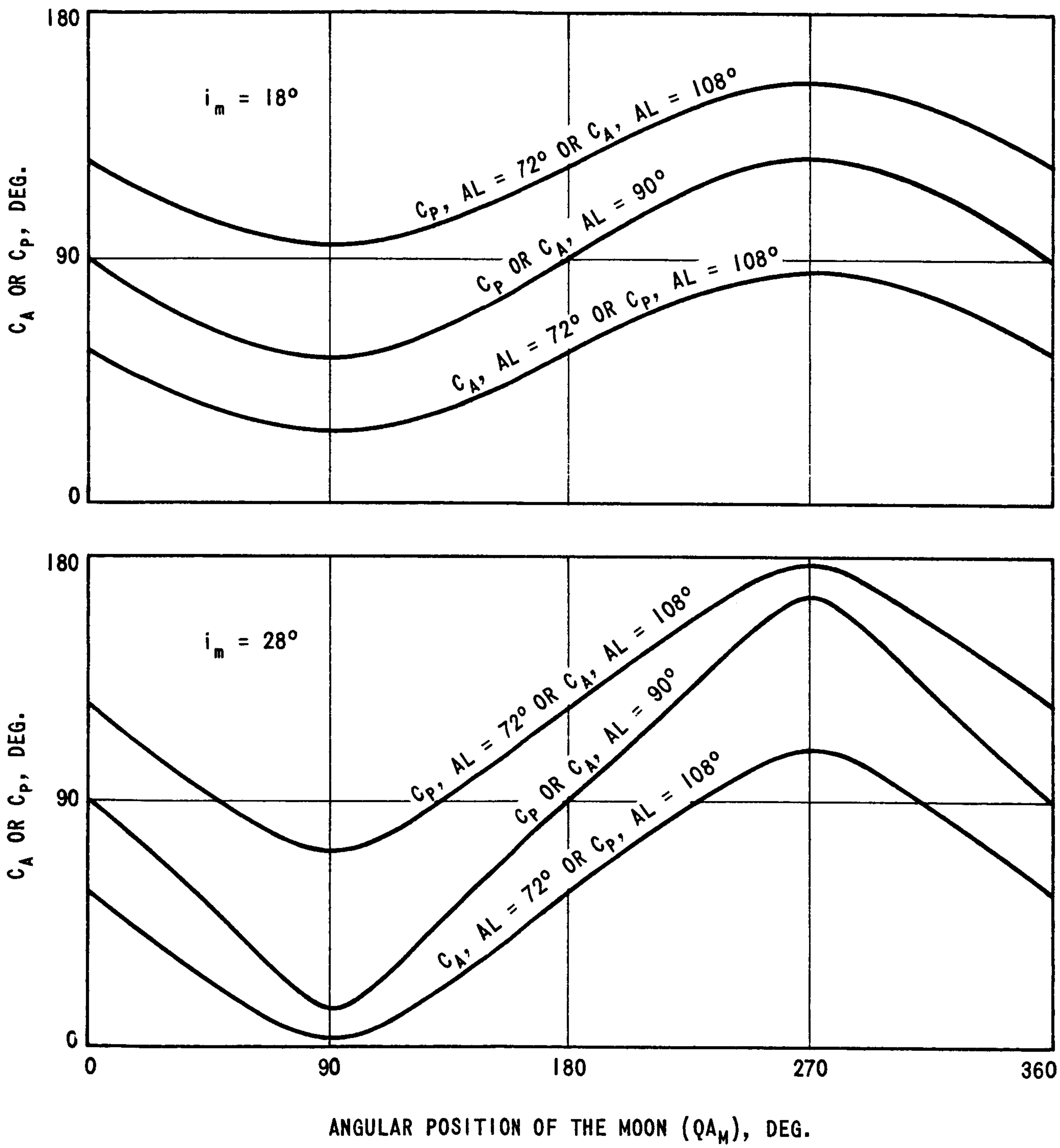

FIGURE 18 - BEHAVIOR OF C_A AND C_P

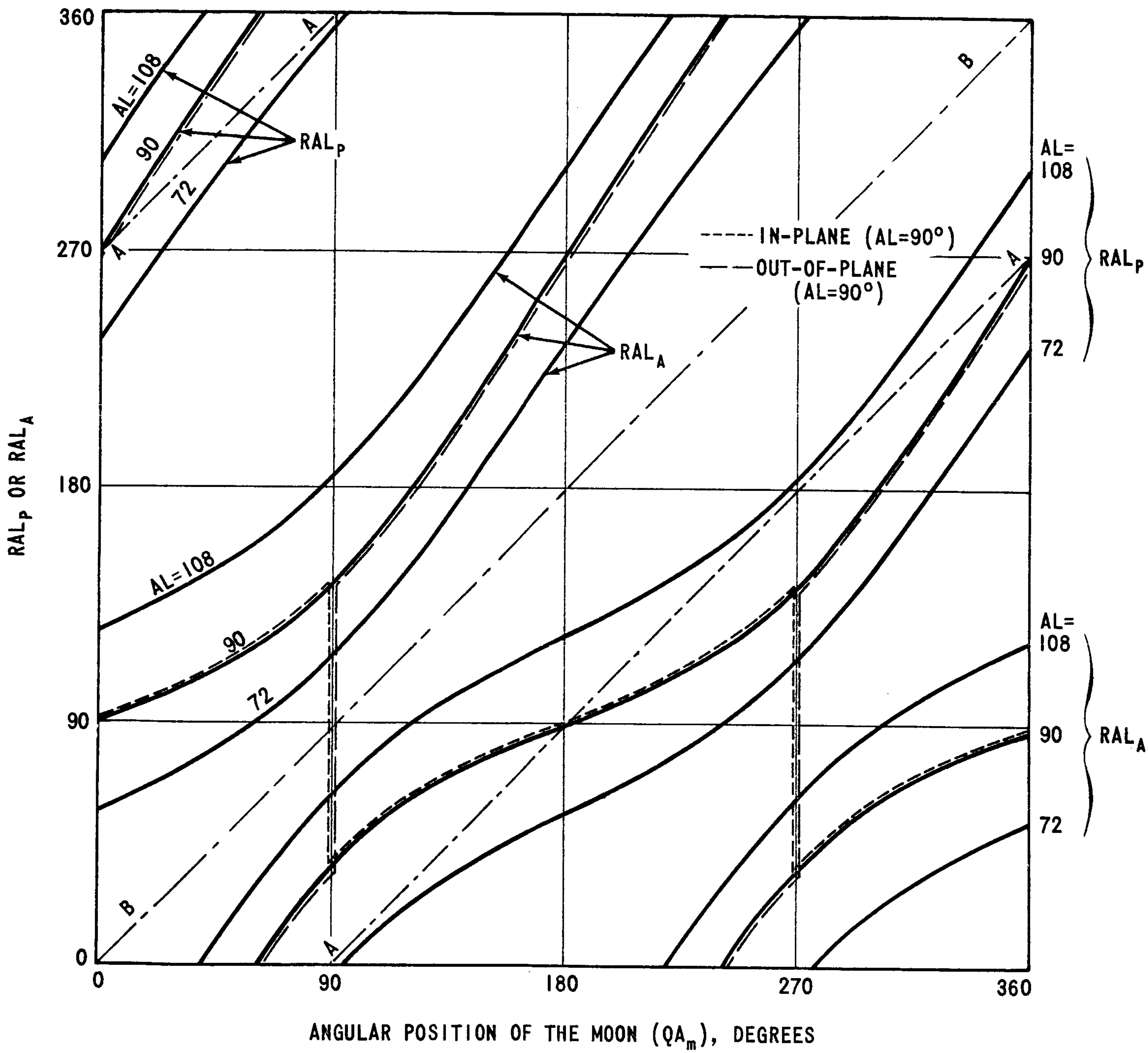

FIGURE 19 - BEHAVIOR OF RAL_A AND RAL_P, i = 18°
(AL = 72°, 90°, AND 108°)

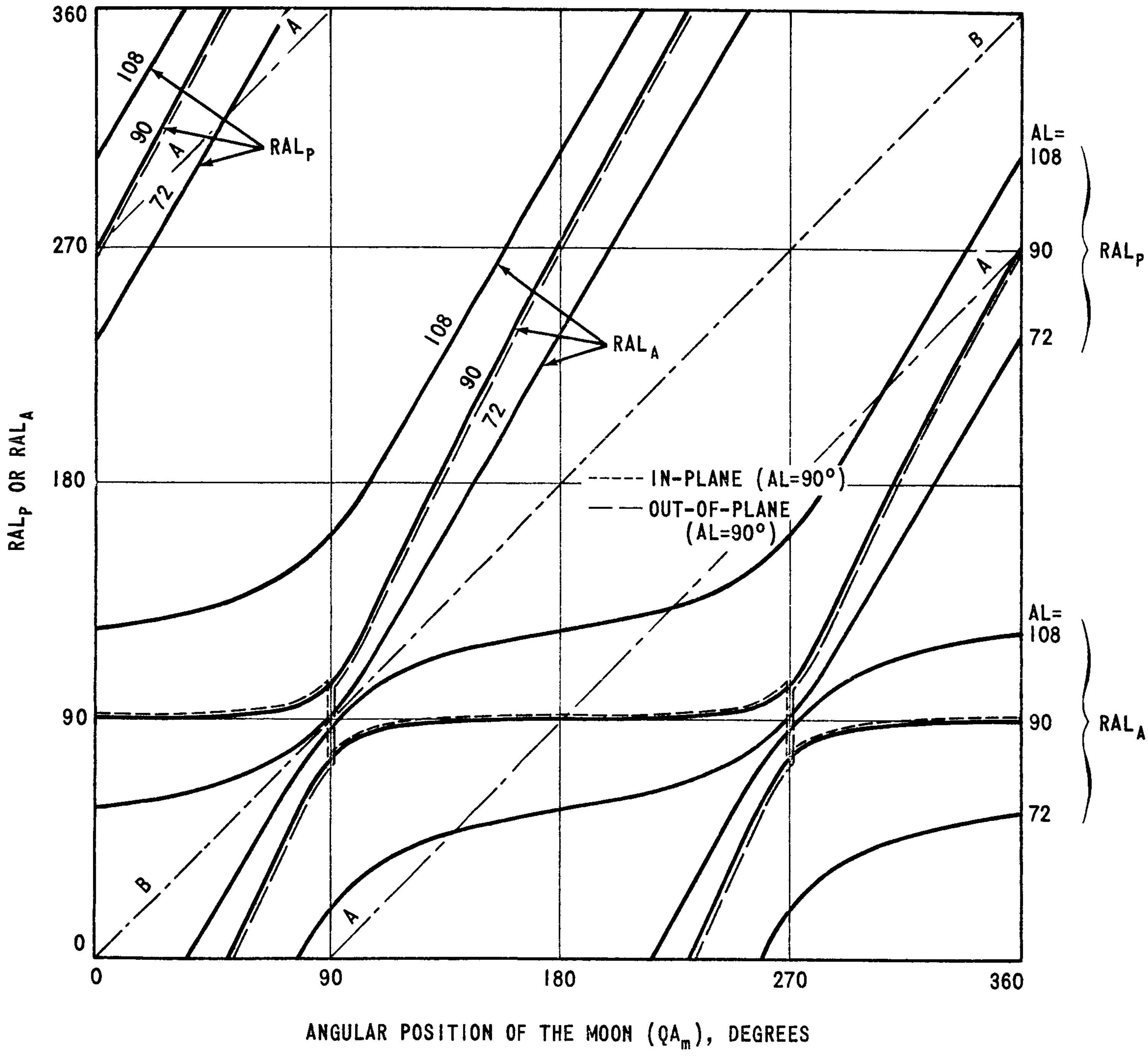

FIGURE 20 - BEHAVIOR OF RAL_P AND RAL_A, $i_m = 28°$ (AL = 72°, 90°, AND 108°)

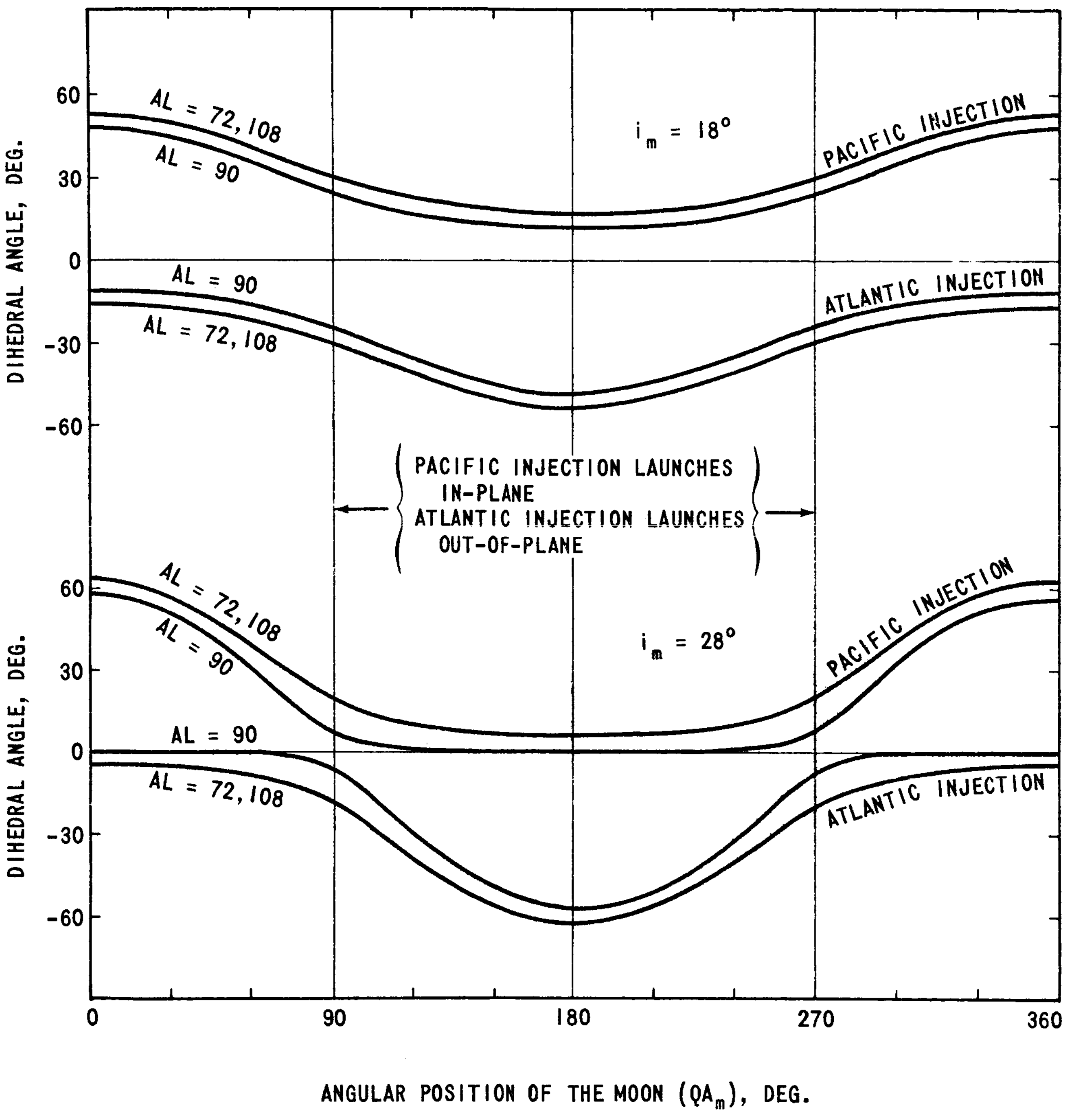

FIGURE 21 - BEHAVIOR OF THE DIHEDRAL ANGLES